

The 26th Annual
ACM International Collegiate
Programming Contest
ASIA Regional - Taejon



Problem C
Modular multiplication of polynomials
Input: poly.in
Output: poly.out

Consider polynomials whose coefficients are 0 and 1. Addition of two polynomials is achieved by 'adding' the coefficients for the corresponding powers in the polynomials. The addition of coefficients is performed by addition modulo 2, i.e., $(0 + 0) \bmod 2 = 0$, $(0 + 1) \bmod 2 = 1$, $(1 + 0) \bmod 2 = 1$, and $(1 + 1) \bmod 2 = 0$. Hence, it is the same as the exclusive-or operation.

$$(x^6 + x^4 + x^2 + x + 1) + (x^7 + x + 1) = x^7 + x^6 + x^4 + x^2$$

Subtraction of two polynomials is done similarly. Since subtraction of coefficients is performed by subtraction modulo 2 which is also the exclusive-or operation, subtraction of polynomials is identical to addition of polynomials.

$$(x^6 + x^4 + x^2 + x + 1) - (x^7 + x + 1) = x^7 + x^6 + x^4 + x^2$$

Multiplication of two polynomials is done in the usual way (of course, addition of coefficients is performed by addition modulo 2).

$$(x^6 + x^4 + x^2 + x + 1)(x^7 + x + 1) = x^{13} + x^{11} + x^9 + x^8 + x^6 + x^5 + x^4 + x^3 + 1$$

Multiplication of two polynomials $f(x)$ and $g(x)$ modulo a polynomial $h(x)$ is the remainder of $f(x)g(x)$ divided by $h(x)$.

$$(x^6 + x^4 + x^2 + x + 1)(x^7 + x + 1) \bmod (x^8 + x^4 + x^3 + x + 1) = x^7 + x^6 + 1$$

The largest exponent of a polynomial is called its degree. For example, the degree of $x^7 + x^6 + 1$ is 7.

Given three polynomials $f(x)$, $g(x)$, and $h(x)$, you are to write a program that computes $f(x)g(x)$ modulo $h(x)$. We assume that the degrees of both $f(x)$ and $g(x)$ are less than the degree of $h(x)$. The degree of a polynomial is less than 1000.

Since coefficients of a polynomial are 0 or 1, a polynomial can be represented by $d+1$ and a bit string of length $d+1$, where d is the degree of the polynomial and the bit string represents the coefficients of the polynomial. For example, $x^7 + x^6 + 1$ can be represented by

8 1 1 0 0 0 0 0 1.

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Input

The input consists of T test cases. The number of test cases (T) is given in the first line of the input file. Each test case consists of three lines that contain three polynomials $f(x)$, $g(x)$, and $h(x)$, one per line. Each polynomial is represented as described above.

Output

The output should contain the polynomial $f(x)g(x)$ modulo $h(x)$, one per line. The output must be written to the file `poly.out`, not to standard output.

Sample Input (poly.in)

```
2
7 1 0 1 0 1 1 1
8 1 0 0 0 0 0 1 1
9 1 0 0 0 1 1 0 1 1
10 1 1 0 1 0 0 1 0 0 1
12 1 1 0 1 0 0 1 1 0 0 1 0
15 1 0 1 0 1 1 0 1 1 1 1 1 0 0 1
```

Output for the Sample Input (poly.out)

```
8 1 1 0 0 0 0 0 1
14 1 1 0 1 1 0 0 1 1 1 0 1 0 0
```